Exam 2
Chapters 2 and 3

Answer the following questions. *Answers without proper evidence of knowledge will not be given credit.* Make sure to make reasonable simplifications. Do not approximate answers. Give exact answers. **No calculators are allowed on this exam.**

True or False (2 points each)

F **1.** $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$

T **2.** $(3 \cdot 4)^2 = 3^2 \cdot 4^2$

F **3.** $(a+b)^2 = a^2 + b^2$

T **4.** $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

F **5.** $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$

F **6.** $\frac{x+4}{x-4} + \frac{x+5}{x+4} = \frac{(x+4)+(x+5)}{(x-4)+(x+4)}$

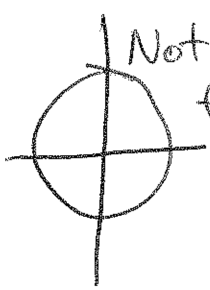
Same question again!!

T **7.** If a function is one-to-one then it is invertible.

T **8.** $(5^4)^2 = (5^2)^4$

Show your work!

1. (9 points) Which of the following represents a function? (Mark all that apply.)
(a) $x^2 + y^2 = 1$ (b) $h = \{(1,2), (5,6), (3,2)\}$ (c) $f(x) = x^3$



Not a function (vertical line test.)

Both functions

2. Let $f(x) = x^2 + 1$ and $g(x) = 3x + 7$. Evaluate the following and simplify. (5 points each)

a) $f \circ g(x)$.

b) $g \circ f(x)$.

a) $f(g(x)) = f(3x+7) = (3x+7)^2 + 1$

$= 9x^2 + 42x + 49 + 1 = 9x^2 + 42x + 50$

b) $g(f(x)) = g(x^2+1) = 3(x^2+1)+7 = 3x^2+3+7$

$= 3x^2 + 10$

3. Let $f(x) = \sqrt{x-3} + 7$, $g(x) = x^3 + 1$ and $h(x) = (x-7)^2$. Evaluate the following functions and find there domains. (You do not need to simplify!) (5 points each)

a) $f + g(x)$.

b) $h \cdot g(3)$.

c) $f/h(x)$.

a) $\sqrt{x-3} + x^3 + 1 + 7$ $\quad dom(f+g) = [3, \infty)$

b) $h \cdot g(x) = (x-7)^2(x^3+1)$; $h \cdot g(3) = (3-7)^2(3^3+1)$

$= (-4)^2(28) = 16 \cdot 28$

$dom(h \cdot g) = \mathbb{R}$

c) $\frac{f}{h}(x) = \frac{\sqrt{x-3}+7}{(x-7)^2}$ $\quad dom(f/h) = [3, 7) \cup (7, \infty)$.

4. Let $j(x) = (x-1)^2 + 3$. Evaluate the following and simplify.

a) $j(t)$ (3 points)

b) $j(3x+1)$ (7 points)

a) $j(t) = (t-1)^2 + 3 = t^2 - 2t + 1 + 3$

$= t^2 - 2t + 4$

b) $j(3x+1) = (3x+1-1)^2 + 3$

$= (3x)^2 + 3$

$= 9x^2 + 3$

5. By completing the square, write $f(x) = x^2 + 6x - 4$ in vertex form and state the vertex. (10 points)

$$f(x) = x^2 + 6x + 9 - 9 - 4$$
$$= (x+3)^2 - 13$$

Vertex $= (-3, -13)$

6. Determine all possible rational zeroes for the polynomial function $h(x) = 3x^5 + 4x^4 - 3x^2 + 15$. (10 points)

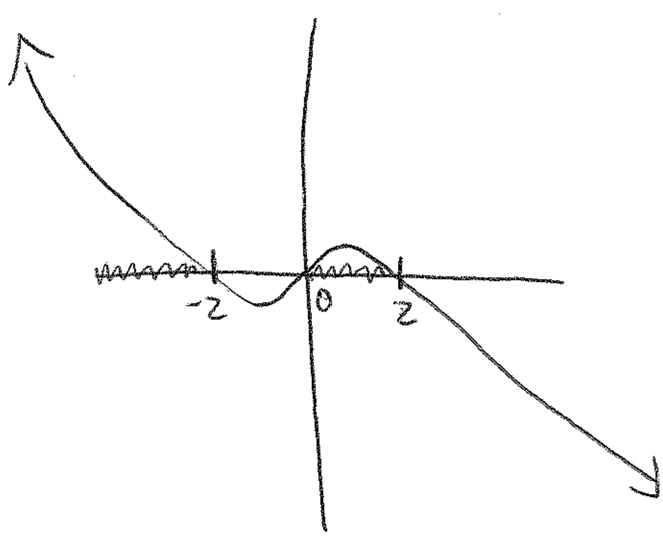
$p = \pm 1, \pm 3, \pm 5, \pm 15$
$q = \pm 1, \pm 3$

$$\frac{p}{q} = \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{3}, \pm \frac{5}{3}$$

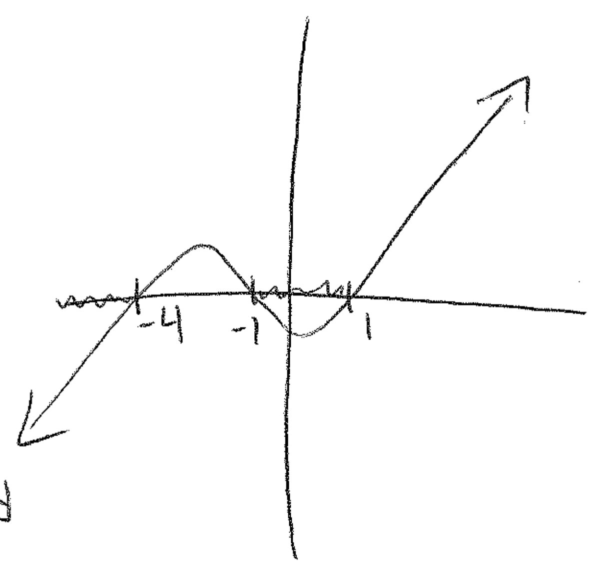
7. Use the graphs given below to solve the following inequalities. (5 points each)

a) $-x^3 \geq -4x$.

b) $x^3 + 4x^2 - x - 4 < 0$.



$-x^3 + 4x$



$x^3 + 4x^2 - x - 4$

a) $(-\infty, -2] \cup [0, 2]$

b) $(-\infty, -4) \cup (-1, 1)$

8. Find the asymptotes of the graph of $\frac{x^2-2x+1}{x-2}$.

Vertical Asymptotes at $x=2$

No Horizontal Asymptotes

Slant Asymptote at $y=x$.

$$\begin{array}{r|rrr} 2 & 1 & -2 & 1 \\ & & 2 & 0 \\ \hline & 1 & 0 & 1 \end{array}$$

$x^2-2x+1=(x-2)x+1$

Extra Credit

EC 1. Determine if $f(x) = \frac{\sqrt{3x-1}+7}{2}$ is invertible. If so, find $f^{-1}(x)$. (5 points)

$f(x_1)=f(x_2)$

$\frac{\sqrt{3x_1-1}+7}{2} = \frac{\sqrt{3x_2-1}+7}{2}$

$\sqrt{3x_1-1}+7 = \sqrt{3x_2-1}+7$

$\sqrt{3x_1-1} = \sqrt{3x_2-1}$

$3x_1-1=3x_2-1$

$3x_1=3x_2$

$x_1=x_2$

Yes 1-1

so invertible

$2f(x)=\sqrt{3x_1-1}+7$

$2f(x)-7=\sqrt{3x_1-1}$

$(2f(x)-7)^2=3x_1-1$

$(2f(x)-7)^2+1=3x_1$

$\frac{(2f(x)-7)^2+1}{3}=x$

So

$f^{-1}(x)=\frac{(2x-7)^2+1}{3}$.

EC 2. Suppose y varies directly as x and inversely as the square of z. If $y = 9$ when $x = 18$ and $z = 3$ then what is the varation constant, k?

$y=k\frac{x}{z^2}$

$9=k\frac{18}{9}$

$\boxed{k=\frac{9}{2}}$